State-dependence of the Zero Lower Bound Government Spending Multiplier

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Abstract

Government spending multipliers under constant, zero nominal interest rates can be either large or small in the standard new Keynesian model, which previous work attributed to differences in the cause and severity of the zero lower bound. This paper shows that the government spending multiplier under constant, zero nominal interest rates is not state-dependent (in the linearized model) or only minimally so (in the nonlinear model). Instead, the fiscal experiment is an important determinant of the government spending multiplier. Previous work has reached different conclusions because it simultaneously changed the zero lower bound experiment together with the fiscal experiment.

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1 Introduction

The size of government spending multipliers under constant, zero nominal interest rates remains a topic of contention. The literature is divided among approaches that yield large government spending multipliers (e.g., Christiano, Eichenbaum, and Rebelo, 2011; Woodford, 2011) or small government spending multipliers (e.g., Boneva, Braun, and Waki, 2016; Mertens and Ravn, 2014), even within the same standard new Keynesian model. These differences are at least in part attributed to differences in the zero lower bound experiment: that the persistence, the severity, or the source (type of shock) of the zero lower bound affects the efficacy of fiscal policy (Woodford, 2011; Christiano et al., 2011; Boneva et al., 2016; Mertens and Ravn, 2014).

By contrast, this paper shows that the efficacy of the same fiscal experiment in the standard new Keynesian model under constant, zero nominal interest rates is largely invariant to the depth, persistence, or the source of the zero lower bound. Instead, variations in the fiscal experiment in these studies help account for wide the differences in government spending multipliers.

The intuition is simple. The linearized standard new Keynesian model is linear *conditional* on constant, zero nominal interest rates. Because of the linearity, any exogenous process that affects this economy while nominal interest rates remain zero will always have the same impact on output and inflation irrespective of the depth of the recession, the source of the zero lower bound, or the remaining duration of the zero lower bound. In the nonlinear version of the standard new Keynesian model these claims are not exact, but the differences are small in most cases. Even when there exist two zero lower bound equilibria in the nonlinear model (under standard selection rules), then the same fiscal policy path has similar government spending multipliers across the two equilibria.

Instead, the persistence of *fiscal policy* under constant nominal interest rates is a central determinant of the government spending multiplier. Increasing the persistence of government spending typically raises the multiplier. However, a threshold can exist (a "bifurcation"), after which greater persistence can cause multipliers to become small and even negative. Some previous work simultaneously changed the persistence of fiscal policy and the zero lower bound, and then erroneously attributed the variation in government spending multipliers to changing the zero lower bound persistence rather than changing the fiscal policy.

This paper thus helps reconcile the wide differences in government spending multipliers under constant, zero nominal interest rates obtained in the literature. A second contribution is to the debate between Boneva et al. (2016) and Christiano, Eichenbaum, and Johannsen (2016) on how to select among multiple (standard) equilibria for policy analysis. My results suggest that, in most cases, equilibrium choice *per se* is not an important determinant of government spending multipliers.

This paper does not provide a complete explanation for why constant nominal interest rate government spending multipliers differ so markedly in the literature. For example, even holding the fiscal experiment fixed, a different equilibrium selection criterion can substantially change the government spending multiplier as emphasized by Cochrane (forthcoming). I follow much of the literature in adopting the minimum state variable criterion to select equilibria (Eggertsson and Woodford, 2003; Mertens and Ravn, 2014; Boneva et al., 2016; Christiano et al., 2016).

Like Woodford (2011), Christiano et al. (2011), Boneva et al. (2016), and Mertens and Ravn (2014), I examine the state dependence of government spending multipliers conditional on constant nominal interest rates. This object is distinct from the government spending multiplier when the fiscal shock changes the expected path of nominal interest rates. An example of this second case is when the government spending shock persists after the zero lower bound ceases to be a binding constraint. Then consumers may expect higher nominal interest rates in the future, which would change the output effects today (e.g., Woodford, 2011; Erceg and Lindé, 2014).

A separate question is why the government spending multiplier under constant, zero nominal interest rates is so sensitive to the specification of the fiscal experiment. I provide a resolution in a companion paper (Wieland, 2018). There I show that the minimum state variable criterion used in the literature implicitly invokes different equilibrium selection rules for different fiscal experiments.

1.1 Literature I briefly highlight studies arguing that the depth or the nature of the zero lower bound affects the government spending multiplier. The point is not to single out particular work, but highlight that these claims are present in leading studies. I do not claim that the mathematical derivations in these studies are incorrect. Rather, a common feature is that these studies change the fiscal experiment along with the depth or persistence of the zero lower bound, thereby conflating the latter change with the former.

First, Woodford (2011, p. 20) argues that "increased government purchases when interest rates are at the zero bound should be a powerful means through which to stave off economic crisis precisely in those cases in which the constraint of the zero lower bound would otherwise be most crippling—namely, those cases in which there is insufficient confidence that the disruption of credit markets will be short-lived." He reaches this conclusion based on an experiment that simultaneously lengthens the expected duration of the zero lower bound and the expected duration of fiscal policy.

Christiano et al. (2011, p. 96 and figure 2) conduct comparative static exercises simultaneously for the government spending multiplier and the output gap. They note that "the government-spending multiplier is particularly large in economies in which the output costs of being in the zero-bound state are very large." This statement is correct for their exercises involving parameters, such as the labor supply elasticity. However, one of their exercise entails changes in both the expected length of the zero lower bound and that of fiscal policy. For this exercise the government spending multiplier is independent of the output loss conditional on fixing the persistence of the government spending shock.

Cochrane (forthcoming, p. 14) notes that "the multipliers increase exponentially as the length of the liquidity trap increases..." This comparative static also increases the length of the government spending shock.

Turning to the second claim, Mertens and Ravn (2014) argue that increasing government purchases crowds out consumption in a liquidity trap caused by a loss of confidence, whereas it crowds in consumption in a liquidity trap caused by a fundamental shock. However, these outcomes are also based on two different fiscal experiments. The persistence of government spending is high in a confidence-driven liquidity trap and low in a fundamental liquidity trap.

Boneva et al. (2016, p. 227) argue that "[t]he expected duration of zero interest rates is crucial for the size of the government purchase multiplier." Here too, the expected duration of fiscal policy changes with the expected duration of zero interest rates.

By contrast, Miyamoto, Nguyen, and Sergeyev (2016) show that government spending multipliers can be large even in a fundamental liquidity trap when the persistence of the fiscal shock is sufficiently short. This paper helps rationalize this result by showing that the persistence of fiscal policy determines the government spending multiplier under constant nominal interest rates.

2 Model

The model is a standard new Keynesian model (Woodford, 2011), in which I follow the implementation by Boneva et al. (2016). Since the model is standard, I only report the first order condition in the text. Appendix A derives these conditions.

Optimal consumption behavior is characterized by an Euler equation,

$$1 = \beta(1 + r_t^n)\mathbb{E}_t \frac{1 + i_t}{\Pi_{t+1}} \frac{C_t}{C_{t+1}}$$

where C_t is consumption, i_t is the net nominal interest rate, Π_t is the gross inflation rate, and r_t^n is the natural rate of interest. Consumption choices are governed by intertemporal substitution. A higher real interest rate induces agents to postpone consumption and vice-versa.

Firms face quadratic price adjustment cost following Rotemberg (1982). Their optimal pricing behavior yields a nonlinear Phillips curve,

$$(\Pi_t - 1)\Pi_t = \kappa^* \left(\frac{C_t Y_t^{\psi}}{Z_t^{1+\psi}} - 1 \right) + \beta(1 + r_t^n)\mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{Y_{t+1}}{Y_t} (\Pi_{t+1} - 1)\Pi_{t+1} \right]$$

where Π_t is gross inflation, Y_t is output, and Z_t is productivity. The parameter κ^* is the elasticity of inflation with respect to real marginal cost, $\frac{C_t Y_t^{\psi}}{Z_t^{1+\psi}}$, and the parameter ψ is the inverse Frisch elasticity. This Phillips curve implies that an increase in current (or expected) marginal cost will increase inflation today.

The resource constraint of the economy is,

$$Y_t = C_t + G_t + \frac{\gamma}{2}(\Pi_t - 1)^2 Y_t$$

where $\frac{\gamma}{2}(\Pi_t - 1)^2 Y_t$ are the resource costs of price adjustment. Like Boneva et al. (2016), I assume that resources used in price adjustment are lost. Alternatively, in section 4 I also conduct experiments when these resources are rebated to households. For the linearized model this distinction is irrelevant, since the quadratic price adjustment cost term drops out in the linearization.

The central bank follows an interest rate rule,

$$1 + i_t = \max\{\beta^{-1} + \phi(\Pi_t - 1), 1\}$$

where $\phi > 1$ governs the response to inflation. Since the (net) nominal interest rate is zero in most of my experiments, this rule serves primarily as an equilibrium selection device.

I first work with the linearized version of the model, which is simpler and contains all the intuition. It consists of the following set of linear difference equations,

$$
\begin{aligned}
c_t &= \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \\
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa^* \{[1 + \psi(1 - s_g)]c_t + s_g \psi g_t - (1 + \psi) z_t\} \\
y_t &= s_g g_t + (1 - s_g) c_t \\
i_t &= \max\{0, \beta^{-1} - 1 + \phi \pi_t\},
\end{aligned}
$$

where lowercase letters indicate log-deviations from steady-state. The parameter s_g is the steady-state government spending share in output.

3 Government spending multipliers in the linearized model

3.1 Shock process I assume that at $t = 0$ there is an unanticipated decrease in the natural rate of interest and a simultaneous unanticipated increase in government spending. These two exogenous variables subsequently follow a three-state Markov process,

State 1:	$r_t^n = \bar{r} < 0,$	$g_t = \bar{g} > 0$
State 2:	$r_t^n = \bar{r} < 0,$	$g_t = 0$
State 3:	$r_t = 0,$	$g_t = 0$

The economy begins in state 1. It transitions to state 2 with probability $p_z - p_g = \text{Prob(State 2|State 1)}$, transitions to State 3 with probability $1 - p_z = \text{Prob(State 3|State 1)}$, and remains in state 1 with probability p_g. In state 2 the natural rate of interest remains low, but government spending returns to steady state. From here, the economy transitions to state 3 with probability $1 - p_z = \text{Prob(State 3|State 2)}$ and remains in state 2 with probability p_z. State 3 features no shock and is absorbing. This structure implies that the natural rate shock persists with probability p_z each period, whereas the government spending shock persists with probability p_g each period.

I chose to a combination of $\bar{r}$ and $\bar{g}$ such that the zero bound will bind in state 1 and state 2.[1] It precludes government spending from being sufficiently large to take the economy out of the zero lower bound. Figure 1 provides a schematic overview in which state the zero lower bound will be binding and in which state the fiscal shock will be active, along with the transition probabilities.

The standard equilibrium selection criteria is to chose an equilibrium that is bounded going forward in time. When $(1-\beta p_z)(1-p_z) > \kappa^*[1+\psi(1-s_g)]p_z$, then this criterion yields a unique equilibrium near the zero-inflation steady state. For the complement set, I follow Mertens and Ravn (2014) and select the (unique) stationary zero lower bound sunspot equilibrium. Thus, for $(1-\beta p_z)(1-p_z) < \kappa^*[1+\psi(1-s_g)]p_z$, I consider the forcing process as,

$$\begin{aligned} &\text{State 1:} && g_t = \bar{g} > 0 \\ &\text{State 2:} && g_t = 0 \\ &\text{State 3:} && g_t = 0 \end{aligned}$$

I assume that the sunspot causes the zero lower bound to bind in state 1 and state 2, and that there are no more sunspots in state 3. As in the case of the fundamental shock, I restrict my attention to government spending shocks that do not lift the economy out of the zero lower bound.[2] Thus, the outcomes for the nominal interest rate and government spending are the same as in figure 1.

To distinguish the two shock processes and associated equilibria, I follow Boneva et al. (2016) and Mertens and Ravn (2014) and call the first "fundamental equilibrium" and the second "sunspot equilibrium."

[1]Specifically, $\bar{g} < \bar{g}^{crit,fund}$, where

$$\frac{\kappa^* s_g \psi(1-p_g)}{(1-\beta p_g)(1-p_g) - \kappa^*[1+\psi(1-s_g)]p_g}\bar{g}^{crit,fund} < -(\beta^{-1}-1) - \frac{(1-\beta p_z)}{(1-\beta p_z)(1-p_z) - \kappa^*[1+\psi(1-s_g)]p_z}\bar{r}$$

[2]Specifically, $\bar{g} < \bar{g}^{crit,sunsp}$, where

$$\frac{\kappa^* s_g \psi(1-p_g)}{(1-\beta p_g)(1-p_g) - \kappa^*[1+\psi(1-s_g)]p_g}\bar{g}^{crit,sunsp} < -(\beta^{-1}-1) - \frac{(1-\beta p_z)}{(1-\beta p_z)(1-p_z) - \kappa^*[1+\psi(1-s_g)]p_z}(\beta^{-1}-1)$$

3.2 Solution: fundamental equilibrium The locally unique forward-bounded equilibrium has a stationary solution for state 1:

$$
\begin{aligned}
c_1 &= \frac{(1-\beta p_z)}{(1-\beta p_z)(1-p_z)-\kappa^*[1+\psi(1-s_g)]p_z}\bar{r} + \frac{\kappa^* s_g \psi p_g}{(1-\beta p_g)(1-p_g)-\kappa^*[1+\psi(1-s_g)]p_g}\bar{g} \\
\pi_1 &= \frac{\kappa^*[1+\psi(1-s_g)]}{(1-\beta p_z)(1-p_z)-\kappa^*[1+\psi(1-s_g)]p_z}\bar{r} + \frac{\kappa^* s_g \psi (1-p_g)}{(1-\beta p_g)(1-p_g)-\kappa^*[1+\psi(1-s_g)]p_g}\bar{g} \\
i_1 &= 0
\end{aligned}
$$

In the fundamental equilibrium, a decrease in the natural rate of interest induces agents to postpone consumption. The reduction in consumption lowers current and expected future marginal costs, causing deflation. The central bank reduces the nominal interest rate to alleviate the recession. Here I assume that the decrease in the natural rate shock is sufficiently large such that the central bank runs into the zero lower bound constraint, $i_t = 0$.

The government spending shock instead raises the marginal cost of production, in turn raising inflation and inflation expectations. At the zero lower bound, this reduces ante real interest rates so that consumption increases through intertemporal substitution (Christiano et al., 2011; Cochrane, forthcoming).

Note that the effect of the natural rate shock is determined by its persistence p_z, whereas the effect of the government spending shock is determined by its persistence p_g. There are no interaction effects between government spending and the natural rate shock.

For state 2 the solution is analogous with $\bar{g} = 0$, and for state 3 the solution is the steady state.

3.3 Solution: sunspot equilibrium The stationary equilibrium for state 1 is:

$$
\begin{aligned}
c_1 &= \frac{(1-\beta p_z)}{(1-\beta p_z)(1-p_z)-\kappa^*[1+\psi(1-s_g)]p_z}(\beta^{-1}-1) + \frac{\kappa^* s_g \psi p_g}{(1-\beta p_g)(1-p_g)-\kappa^*[1+\psi(1-s_g)]p_g}\bar{g} \\
\pi_1 &= \frac{\kappa^*[1+\psi(1-s_g)]}{(1-\beta p_z)(1-p_z)-\kappa^*[1+\psi(1-s_g)]p_z}(\beta^{-1}-1) + \frac{\kappa^* s_g \psi (1-p_g)}{(1-\beta p_g)(1-p_g)-\kappa^*[1+\psi(1-s_g)]p_g}\bar{g} \\
i_1 &= 0
\end{aligned}
$$

The sunspot shock is such that agents anticipate higher real interest rates in the future, which induces them to reduce consumption. The ensuing decline in marginal cost induces deflation, which validates believes of higher real rates when the central bank becomes constrained by the zero lower bound (Mertens and Ravn, 2014).

The effects of government spending depend on the magnitude of p_g. When the persistence of the fiscal experiment is sufficiently small, $(1-\beta p_g)(1-p_g) > \kappa^*[1+\psi(1-s_g)]p_g$, then government spending raises consumption and inflation analogous to the fundamental equilibrium. However, when $(1-\beta p_g)(1-p_g) < \kappa^*[1+\psi(1-s_g)]p_g$, higher government spending reduces consumption because consumers anticipate higher real interest rates. The decline in consumption is sufficiently large such that the marginal cost of production falls. This worsens the deflation, which validates the consumer expectations of higher real interest rates at the zero lower bound.

Some properties of the fundamental solution equally apply to the sunspot solution: First, there is no interaction of the sunspot shock with the fiscal shock. Second, the output and inflation effects of fiscal policy are governed by the persistence of fiscal policy p_g, not that of the sunspot shock p_z.

The solution for state 2 is analogous with $\bar{g} = 0$, and in state 3 the economy is in the steady state.

3.4 Government spending multiplier The government spending multiplier is the same in both equilibria given an identical fiscal experiment,

$$\begin{aligned}\mu_g^{ZLB} &= \frac{dy_t}{d\bar{g}}\frac{1}{s_g} = 1 + \frac{dc_t}{d\bar{g}}\frac{1}{s_g} \\ &= 1 + \frac{\kappa^*\psi p_g}{(1-\beta p_g)(1-p_g) - \kappa^*[1+\psi(1-s_g)]p_g}\end{aligned} \tag{1}$$

That is, irrespective of whether the zero lower bound is caused by a sunspot shock or a fundamental shock, the government spending multiplier is identical given for the same government spending process. Further, the government spending multiplier is independent of the expected duration of the zero lower bound. Figure 2a plots a numerical example. Since the expected durations is one determinant of the recession depth, it follows that the government spending multiplier is also independent of the output gap.

Instead, the persistence of government spending, p_g, is a key determinant of the multiplier. Figure 2b plots an example of this relationship. Raising the persistence increases the multiplier until a bifurcation point upon which it becomes negative. Thus, increasing government spending is effective at raising output when p_g is low, but can even lower output when p_g is high.[3]

For low persistence p_g, the government spending shock raises expected inflation, thus lowers real interest rates and raises consumption through intertemporal substitution. Raising p_g strengthens these effects, as expected marginal costs are projected to be higher in the future raising inflation even more today. The snowballing culminates at the bifurcation point where output gains approach an asymptote. On the other side of the bifurcation point the stationary equilibrium features small government spending multipliers.[4] In this case, the government spending shock causes deflation and raises real interest rates, so consumption declines through intertemporal substitution. Consumption falls sufficiently to depress marginal costs, which validates the deflation.

3.5 Intuition The linear model allows for a simple intuition for why the characteristics of the zero lower bound episode do not matter. So long as nominal interest rates do not vary with g_t, the model is completely linear in g_t. Thus, the solution is separable in the fundamental/sunspot shocks and the government spending shock. We can denote the solution for consumption, inflation, and inflation in the absence of government spending shocks as $\hat{c}, \hat{\pi}, \hat{y}$. Define $\Delta x_t = x_t - \hat{x}_t$ as the difference of this solution with the full solution. Then,

[3]Some equilibrium selection criteria do not pick an equilibrium the sunspot equilibrium region, which rules out the case where the government spending multiplier is negative. However, non-existence in part of the parameter space may be undesirable, particularly since Aruoba, Cuba-Borda, and Schorfheide (2016) argue that it is empirically relevant for Japan. I do not need to take a stand on this debate, since my results on state dependence of the government spending multiplier apply equally to the subset of the parameter space in which only the fundamental equilibrium exists.

[4]See Wieland (2018) for an explanation of the bifurcation result.

we difference the model equations for the solution with and without government spending shocks,

$$\begin{aligned}
\Delta c_t &= \mathbb{E}_t \Delta c_{t+1} + \mathbb{E}_t \Delta \pi_{t+1} \\
\Delta \pi_t &= \beta \mathbb{E}_t \Delta \pi_{t+1} + \kappa^* \left\{ [1 + \psi(1 - s_g)] \Delta c_t + s_g \psi g_t \right\} \\
\Delta y_t &= s_g g_t + (1 - s_g) \Delta c_t \\
\Delta i_t &= 0
\end{aligned}$$

We can solve this system without any reference as to what generates the zero lower bound, be it a fundamental shock or a sunspot shock (so long as $\Delta i_t = 0$ while g_t is nonzero). It then follows that the solution for the government spending multiplier is independent of the nature of the zero lower bound regime.

3.6 Evaluation These results appear to contradict existing studies surveyed in section 1.1, which have argued that the sign and size of the government spending multiplier depends on the severity and source of the zero lower bound. The main difference is that these papers constrain the persistence of the zero lower bound and government spending to be the same, $p = p_g = p_z$. Then, raising p in the fundamental case increases both the severity and duration of the zero lower bound as well as increase the government spending multiplier. And for sufficiently high p we enter the sunspot case, where the government spending multipliers are small and possibly even negative.

But these comparative statics on p simultaneously change the nature and severity of the zero lower bound *as well as the fiscal experiment under consideration.* As my results show, it is the latter comparative static, not the former, that accounts for the differences in the government spending multiplier.

4 Nonlinear model

In the linearized new Keynesian model, the effect of the government spending shock under constant, zero nominal interest rates is always independent of the duration and depth of the zero lower bound. However, in the nonlinear version of the model I can make no such absolute statement. Instead, it is a quantitative question whether government spending multipliers are largely independent of the depth and duration of the zero lower bound.

I address three distinct issues. First, does the multiplier vary significantly with the type of shock generating the recession holding fixed its impact on output and inflation? Second, does the size of the recession significantly change the size of the government spending multiplier? Third, if the nonlinear model has multiple zero lower bound equilibria under the standard equilibrium selection criterion, then is the government spending multiplier sensitive to the equilibrium choice? I argue that in the standard nonlinear new Keynesian model the answer to all three questions is no for most cases. Instead, as in the linear model, the persistence of government spending is a central determinant of the multiplier.

These findings are distinct from Eggertsson and Singh (2016), who compare government spending multipliers in a linear model and a nonlinear model for a given experiment and show that the difference is small. Instead, I compare government spending multipliers across fiscal or zero lower bound experiments within the linear model (above) and now the nonlinear model. Thus, I show that the *sensitivity* of the government spending multiplier to the fiscal or zero lower bound experiment in the nonlinear model is similar to that in the linear model.

I use the nonlinear equations in section 2 and adopt the parameter values from Boneva et al. (2016), $\kappa^* = 0.01546$, $\psi = 0.37$, $\beta = 0.997$, $\gamma = 495.6$. I also impose that government spending is 20% of steady state output, $s_g = 0.2$. The nonlinear model requires taking a stand on whether one computes multipliers for gross output, $\frac{dY_t}{dG_t}$, or multipliers for GDP, $\frac{d(C_t+G_t)}{dG_t}$; the difference being the price adjustment costs. In what follows I always report multipliers for GDP. This statistic directly captures whether consumption increases or decreases with government spending. Leading empirical studies also estimate multipliers for GDP (e.g., Auerbach and Gorodnichenko, 2012; Ramey and Zubairy, 2016).

As before, the shock is a three-state Markov process. I follow Boneva et al. (2016) and also allow for a shock to productivity Z_t in the nonlinear model,

$$\begin{array}{llll} \text{State 1:} & r_t^n = \bar{r}, & Z_t = \bar{Z}, & G_t = \bar{G} \times 1.0001 \\ \text{State 2:} & r_t^n = \bar{r}, & Z_t = \bar{Z}, & G_t = \bar{G} \\ \text{State 3:} & r_t = 0, & Z_t = 1, & G_t = \bar{G} \end{array}$$

The economy begins in state 1. It transitions to state 2 with probability $p_z - p_g = \text{Prob(State 2|State 1)}$, transitions to state 3 with probability $1 - p_z = \text{Prob(State 3|State 1)}$ and remains in state 1 with probability p_g. In state 2 the economy transitions to state 3 with probability $1 - p_z = \text{Prob(State 3|State 2)}$ and remains in state 2 with probability p_z. State 3 is absorbing. In words, the natural rate and productivity shocks persists with probability p_z, whereas the government spending shock persists with probability p_g.

4.1 Varying the type of shock causing the zero lower bound Like Boneva et al. (2016), I look for combinations of $\bar{r}$ and $\bar{Z}$ to hit their Great Recession targets: a 7% drop in GDP and a 1% drop in annual inflation. That is, I fix the depth of the recession and then examine whether the persistence of the zero lower bound affects the size of the government spending multiplier. As emphasized by Boneva et al. (2016), this procedure selects a unique (and empirically relevant) equilibrium conditional on values for p_g and p_z.

Figure 3a plots the government spending multiplier when the persistence of the zero lower bound varies from $p_z = 0.8$ to $p_z = 1$ and the persistence of government spending is fixed at $p_g = 0.8$. The government spending multiplier is essentially flat, although numerically it does decline from 1.18 to 1.14. Thus, raising the expected duration of the zero lower bound from 5 quarters to infinity only has a minimal impact on the government spending multiplier. I conclude that knowing the persistence of the zero lower bound conveys little information on the size of government spending multipliers conditional on the depth of the recession.

Figure 3b instead shows how the government spending multiplier varies with the persistence of the fiscal experiment p_g. Throughout I fix $p_z = 1$ and chose the corresponding shocks, $\bar{r}$ and $\bar{Z}$, to hit the Great Recession targets. This figure highlights the strong dependency of government spending multipliers on p_g, just like in the linear model (figure 2b).

Boneva et al. (2016) reached a different conclusion, because they simultaneously changed the persistence of the zero lower bound, p_z, and of fiscal policy, p_g. They then attributed the change in government spending multipliers to the change in p_z instead of to the change in p_g. Nevertheless, a key conclusion from Boneva et al. (2016) remains intact: Except for a small region to the left of the bifurcation point, the government spending multiplier is below 1.05.

4.2 Varying the severity of the zero lower bound episode Next, I consider the possibility that the nonlinearities of the new Keynesian model generate government spending multipliers that are very different depending on the depth of the recession. In figure 3b, I also plot the government spending multiplier for the Great Depression targets in Boneva et al. (2016). These are a 30% decline in GDP and a 10% decline in inflation. The main difference for these two calibrations is that the bifurcation point is shifted to the right. Away from the two bifurcation points, the government spending multipliers are similar. For example, at $\rho_g = 0.7$ the difference in multipliers across the two calibrations is 0.035.

The shift in the bifurcation point is almost entirely caused by the increased price adjustment costs in the economy-wide resource constraint. In the Great Recession calibration the resource cost is $\frac{\gamma}{2}(\Pi_t - 1)^2 = \frac{495.6}{2}(0.99^{\frac{1}{4}} - 1)^2 = 0.16\%$ of output, whereas in the Great Depression calibration it is $\frac{\gamma}{2}(\Pi_t - 1)^2 = \frac{495.6}{2}(0.9^{\frac{1}{4}} - 1)^2 = 16.7\%$ of output. Figure 4 plots the government spending multipliers when these resource costs are returned lump-sum to households rather than thrown away in the process of price-adjustment. Now the government spending multipliers are essentially identical in the Great Recession and Great Depression scenario. Note that this change preserves the nonlinear structure of the model, but only changes the distribution of the resource cost of price adjustment.

Thus, for a wide range of fiscal experiments varying in persistence p_g, the government spending multiplier is essentially independent of the depth of the recession in the nonlinear new Keynesian model. This extends to essentially the entire range of p_g when resource costs of price adjustment are rebated lump-sum. Government spending multipliers are instead only sensitive to the depth of the recession when p_g is very close to the bifurcation point. For example, when $p_g = 0.865$ then the government spending multiplier is 4.29 in the Great Recession calibration and 1.80 in the Great Depression calibration. (The bifurcation point occurs at $p_g = 0.87$ in the Great Recession calibration.)

4.3 Multiple equilibria As shown by Boneva et al. (2016), the nonlinear new Keynesian model may have multiple minimum state variable equilibria. Again, I ask if the government spending multiplier is very sensitive to the choice of equilibrium, and/or if the fiscal experiment is an important determinant of the government spending multiplier.

For the Boneva et al. (2016) calibration, the range of p_z over which multiple equilibria can occur varies with whether price adjustment costs are rebated or not. If they are not rebated as in Boneva et al. (2016), then two equilibria exist in the range $p_z \in [0.862, 0.892]$, which includes the bifurcation point. Of course, in each case only one equilibrium attains the Great Recession targets. (The equilibrium plotted in figure 3a.) I call this equilibrium "targeted", and I call the other equilibrium "non-targeted." For the case where the adjustment costs are rebated I could not find multiple equilibria for any p_z.

I chose $p_z = 0.87$ as a baseline for illustration. Figure 5a plots the constant, zero nominal interest rate government spending multipliers in the two equilibria when price adjustment costs are not rebated. For much of the range of p_g the multipliers are very similar. The exception is that the bifurcation point in the non-targeted equilibrium is shifted to the right relative to the targeted equilibrium. Note that both Boneva et al. (2016) and Christiano et al. (2016) find large differences in multipliers across equilibria, but they restrict attention to the case where $p_g = p_z \equiv p$. Since multiplicity occurs when p is near the bifurcation point, they automatically focus on the specific region of the parameter space where differences may be large. For example, when $p_g = p_z = 0.87$, then the multiplier is -14.7 in the targeted equilibrium and 2.0 in the non-targeted equilibrium.

But figure 5a still leaves open whether the (potentially) large differences in multipliers near the bifurcation point are due to equilibrium multiplicity per se. I provide two pieces of evidence against this view. First, I calculate government spending multipliers for a calibration that replicates GDP and inflation (14.6% and -14.2%) in the non-targeted equilibrium for $p_z = 0.90$. This equilibrium is unique, and, as shown in figure 5a, yields government spending multipliers are very close to those in the non-targeted equilibrium.

Second, in figure 5b I plot the government spending multipliers for the case $p_z = 0.863$. In this case there are also two equilibria. But, unlike for $p_z = 0.87$, the level of deflation in the two $p_z = 0.863$ equilibria is very similar at -1.4%. As a result, both equilibria also have similar price adjustment costs. As figure 5b shows, the government spending multipliers in the two equilibria are essentially identical.

Thus, multiplicity per se does not appear to have an important quantitative impact on the government spending multiplier. Instead, the differences in multipliers appear to be due to differences in the initial levels of output and inflation just as in figure 3a. And similar to that case, the large price adjustment costs account for much of the differences in government spending multipliers by shifting the bifurcation point. Indeed, rebating the adjustment costs lump-sum eliminates these differences as the non-targeted equilibrium ceases to exist.

4.4 Summary I conclude that the insights from the linear model carry over to the standard nonlinear new Keynesian model in most cases. That is, the new Keynesian government spending multiplier at constant, zero nominal interest rates is primarily determined by the fiscal experiment as opposed to the depth or the source of the zero lower bound.

5 Conclusion

This paper clarifies the state-dependence of the government spending multiplier under constant, zero nominal interest rates in a standard new Keynesian model. The persistence of fiscal policy emerges as a key determinant of government spending multipliers in the model. By contrast, the depth or nature of the zero lower bound are much less important determinants of government spending multipliers (and in many cases even irrelevant). Some existing work has reached different conclusions, because it simultaneously varied the nature of the zero lower bound as well as the fiscal policy under consideration, and then erroneously attributed variation in government spending multipliers to the former change as opposed to the latter change.

I have not provided an explanation for why the fiscal experiment has such a large and discontinuous impact on the government spending multiplier in the standard new Keynesian model. That question is taken up in Wieland (2018). There I show that the minimum state variable criterion used in the literature implicitly invokes different equilibrium selection rules for different fiscal experiments.

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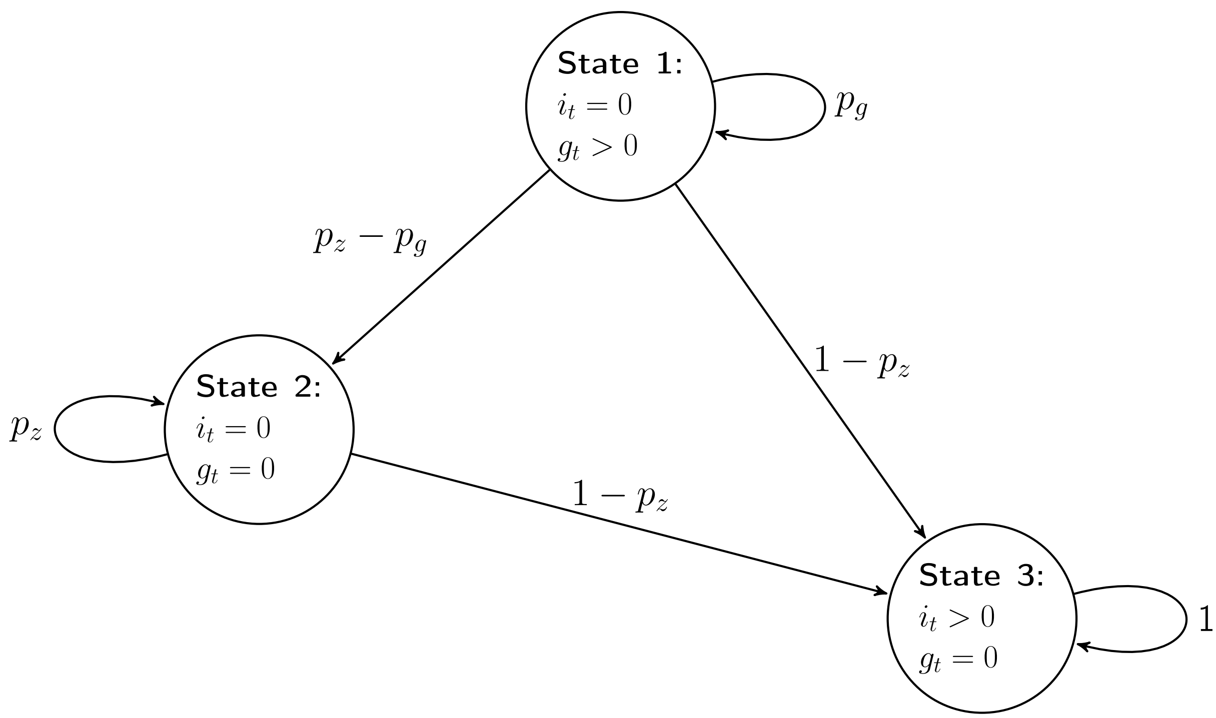
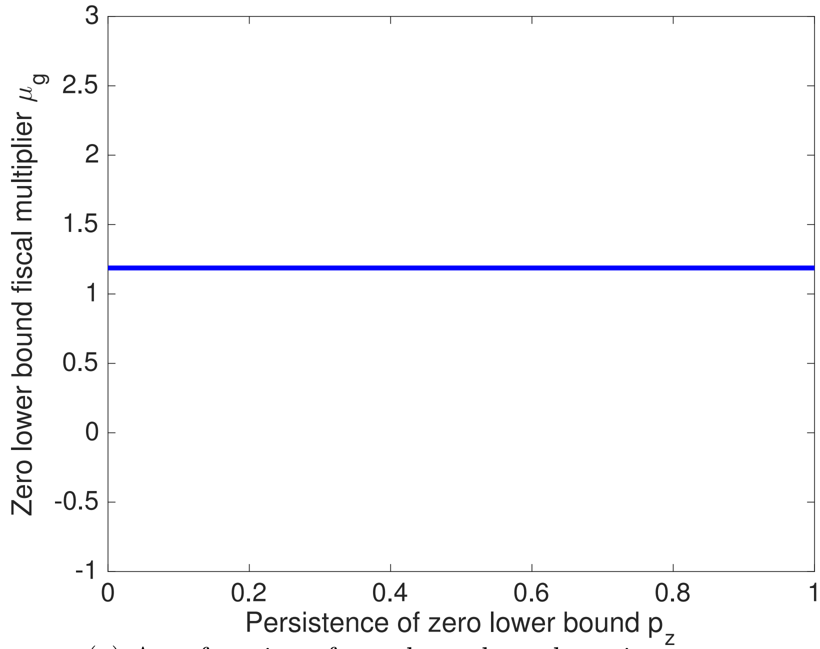
Figure 1 – Outcomes in the three states

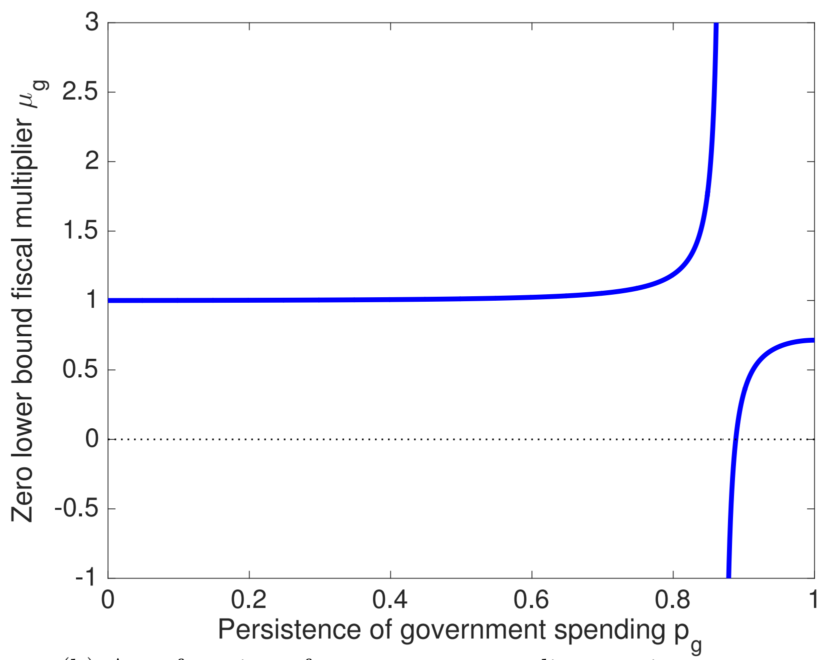


Notes: Overview of outcomes in the three states and transition probabilities. Arrow labels denote the transition probabilities between the states. The outcome $i_t = 0$ implies that the zero lower bound will be binding in the corresponding state, whereas $i_t > 0$ implies that the zero lower bound will not be binding. The outcome $g_t > 0$ shows that a fiscal shock is active in the corresponding state, whereas $g_t = 0$ implies that government spending is at its steady state value.

Figure 2 – Government spending multipliers under constant, zero interest rates in the (linearized) standard new Keynesian model



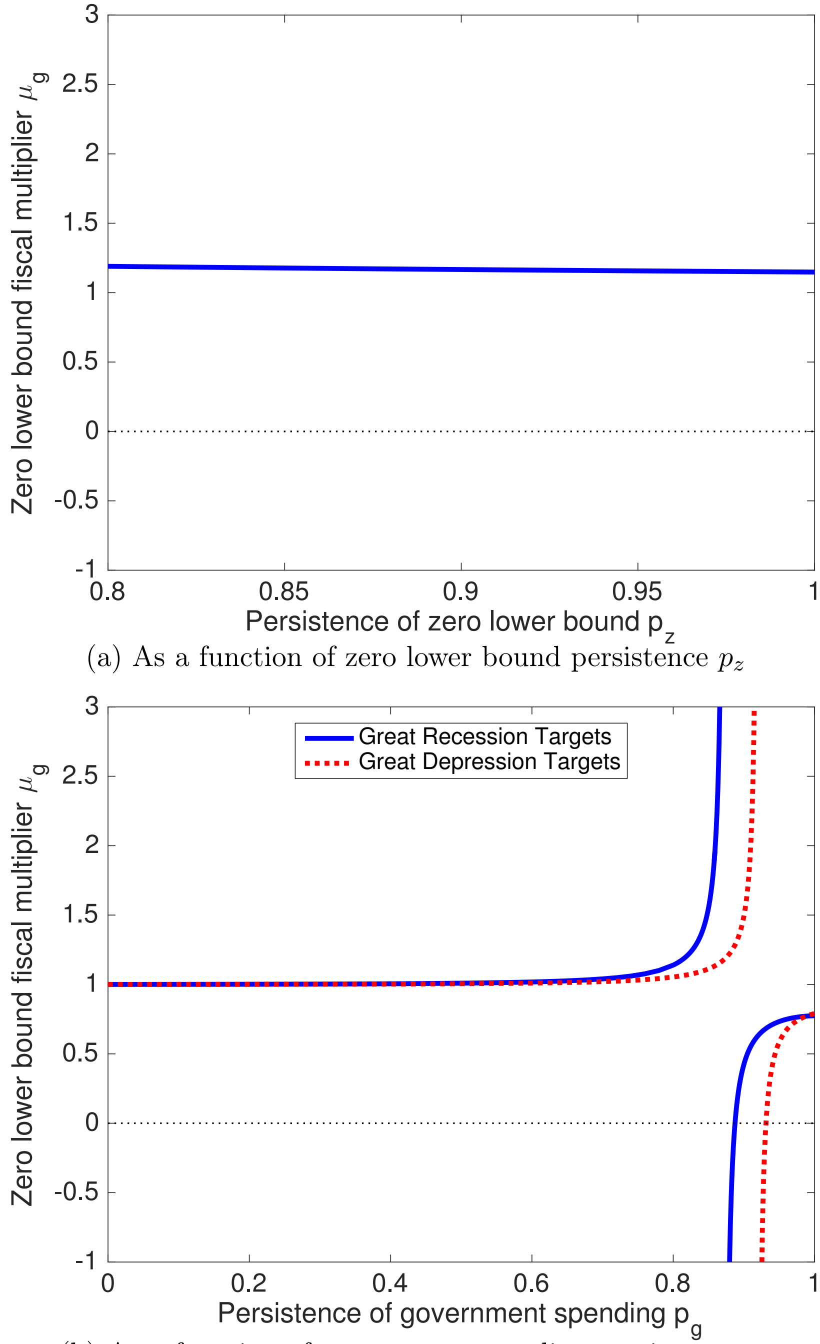
(a) As a function of zero lower bound persistence p_z



(b) As a function of government spending persistence p_g

Notes: The government spending multiplier under constant, zero nominal interest rates (1) is plotted as a function of the zero lower bound persistence p_z (panel (a)), and as a function of the government spending persistence p_g (panel (b)). Parameter values are $\psi = 0.37$, $\beta = 0.997$, $\kappa^* = 0.01547$, $s_g = 0.2$. In panel (a) government spending persistence is fixed at $p_g = 0.8$ and in panel (b) the zero lower bound persistence is fixed at $p_z = 1$.

Figure 3 – Government spending multipliers under constant, zero interest rates in the (non-linear) standard new Keynesian model

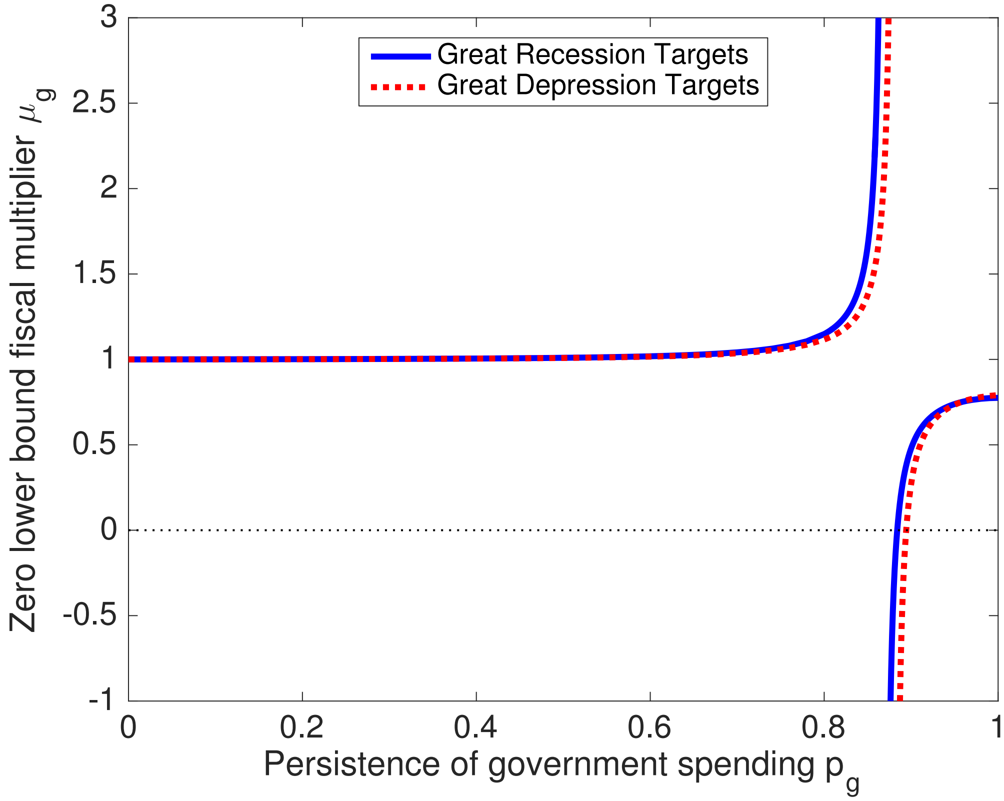


(a) As a function of zero lower bound persistence p_z

(b) As a function of government spending persistence p_g

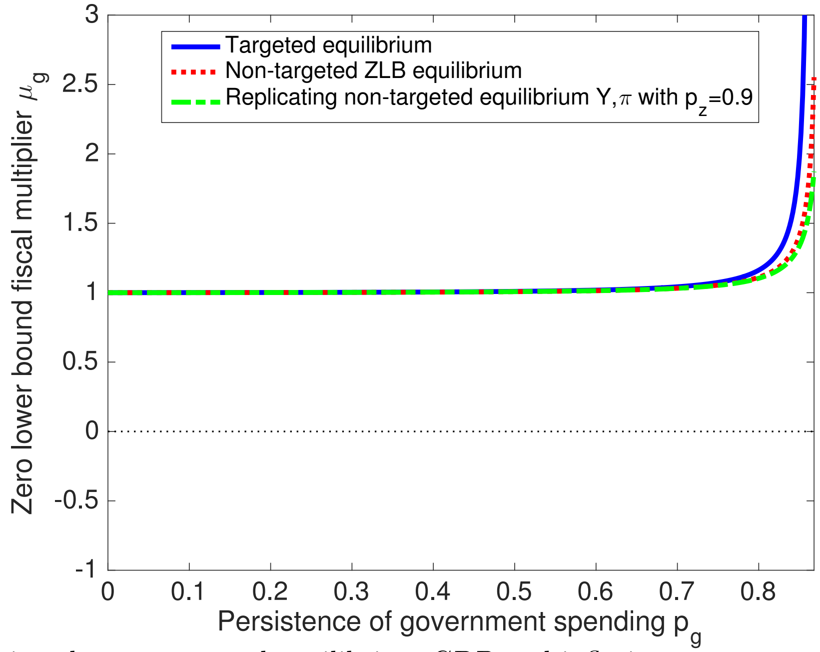
Notes: The government spending multiplier under constant, zero nominal interest rates in the nonlinear standard new Keynesian model is plotted as a function of the zero lower bound persistence p_z (panel (a)), and as a function of the government spending persistence p_g (panel (b)). Parameter values are $\kappa^* = 0.01546$, $\psi = 0.37$, $\beta = 0.997$, $\gamma = 495.6$, $p_g = 0.8$. In panel (a) government spending persistence is fixed at $p_g = 0.8$ and in panel (b) the zero lower bound persistence is fixed at $p_z = 1$. Productivity and natural rate of interest shocks are chosen to hit GDP and inflation targets absent the government spending shock. Great Recession targets are a 1% drop in inflation and a 7% drop in GDP. Great Depression targets are a 10% drop in inflation and a 30% drop in GDP.

Figure 4 – Government spending multipliers under constant, zero interest rates when resource costs are rebated lump-sum in the (nonlinear) standard new Keynesian model

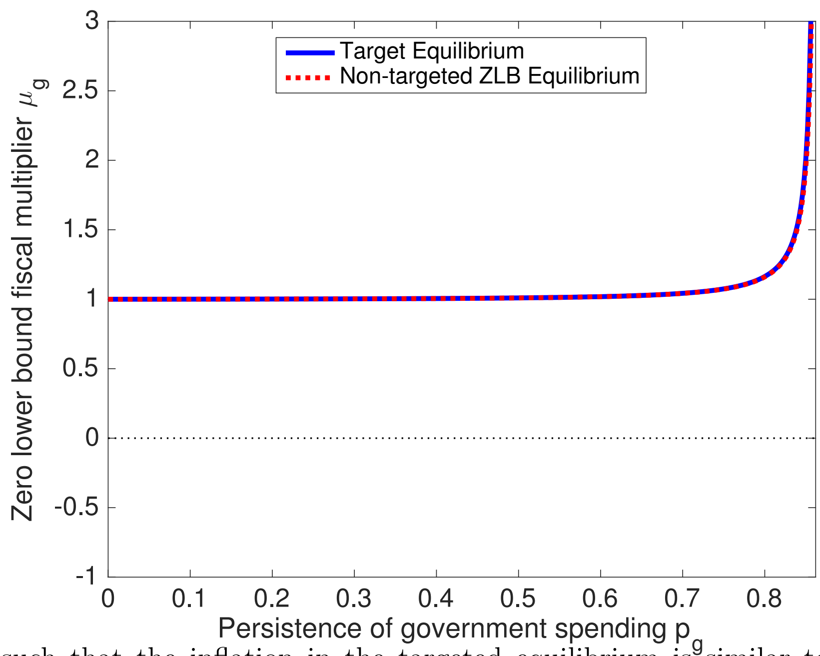


Notes: The government spending multiplier under constant, zero nominal interest rates when resource costs are rebated lump-sum in the nonlinear standard new Keynesian model is plotted as a function of the fiscal shock persistence p_g. Parameter values are $\kappa^* = 0.01546$, $\psi = 0.37$, $\beta = 0.997$, $\gamma = 0$, $p_z = 1$. Productivity and natural rate of interest shocks are chosen to hit GDP and inflation targets. Great Recession targets are a 1% drop in inflation and a 7% drop in GDP. Great Depression targets are a 10% drop in inflation and a 30% drop in GDP.

Figure 5 – Government spending multipliers under constant, zero interest rates when there are two (standard) equilibria in nonlinear standard new Keynesian model



(a) Replicating the non-targeted equilibrium GDP and inflation outcomes when $p_z = 0.9$



(b) Setting p_z such that the inflation in the targeted equilibrium is similar to inflation in the non-targeted equilibrium.

Notes: The government spending multiplier under constant, zero nominal interest rates in the nonlinear standard new Keynesian model is plotted as a function of the fiscal shock persistence p_g. Common parameter values are $\kappa^* = 0.01546$, $\psi = 0.37$, $\beta = 0.997$, $\gamma = 495.6$. The persistence of the zero lower bound is $p_z = 0.87$ in panel (a) and $p_z = 0.863$ in panel (b). Productivity and natural rate of interest shocks are chosen to hit the Great Recession GDP and inflation targets, which are attained in the targeted equilibrium. The non-targeted equilibrium does not attain the targets but is otherwise admissible under the standard selection criterion. In panel (a) the replication of the non-targeted equilibrium attains the same values for GDP and inflation for $p_z = 0.90$. In panel (b) the two equilibria have similar levels of inflation for $p_z = 0.863$.

A Model

A.1 Household Households maximize utility, which is separable preferences over consumption C_t and labor supply L_t,

$$\mathcal{U}_0 = \max \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) = \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\ln C_t - \chi \frac{L_t^{1+\psi}}{1+\psi} \right],$$

where ψ is the inverse Frisch elasticity. ξ_t is a stochastic intertemporal utility shifters. Utility is maximized subject to the period-by-period budget constraints,

$$\lambda_t : \quad B_t + P_t C_t = (1+i_t)B_{t-1} + W_t L_t + \Pi_t - T_t, \qquad \forall t \geq 0$$

where λ_t is the Lagrange multiplier on the budget constraint, B_t are one-period nominal bonds, P_t is the nominal price of real consumption, i_t is the nominal interest rate, W_t is the common nominal wage rate across firms, Π_t are profits remitted by firms, and T_t are lump-sum taxes imposed by the government.

First order conditions for the households are as follows:

$$\begin{aligned} C_t &: \ \xi_t C_t^{-1} = \lambda_t P_t, \\ L_t &: \ \chi \xi_t L_t^{\psi} = \lambda_t W_t, \\ B_t &: \ \lambda_t = \beta \mathbb{E}_t \lambda_{t+1} (1+i_t). \end{aligned}$$

The Euler equation in the text obtains by combining the first and third equation,

$$1 = \beta(1+r_t^n)\mathbb{E}_t \frac{1+i_t}{\Pi_{t+1}} \frac{C_t}{C_{t+1}}$$

where $(1+r_t^n) = \xi_{t+1}/\xi_t$.

A.2 Firms Firms produce varieties indexed by i over the unit interval. Aggregate consumption is a CES aggregate over individual varieties with elasticity of substitution σ,

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

which implies the aggregate price index,

$$P_t = \left[\int_0^1 C_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}},$$

and relative demands,

$$C_t(i) = C_t \left[\frac{P_t(i)}{P_t} \right]^{-\sigma},$$

A similar demand equations exist for the government. In equilibrium, output produced must equal output demanded,

$$C_t(i) + G_t(i) = Y_t(i),$$

Firms produce this output using labor N_t,

$$Y_t(i) = Z_t N_t$$

where Z_t is a stochastic productivity shifter. An employment subsidy $\tau = \frac{1}{\sigma}$ offsets the distortions from monopolistic competition.

Price setting is subject to Rotemberg pricing frictions (Rotemberg, 1982). For each firm, the cost of price adjustment is $\frac{\gamma}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t Y_t$. The optimal reset prices solve the following optimization problem:

$$\max_{\{P_t(i)\}_t} \sum_{t=0}^{\infty} \theta^s Q_{0,t} \left[P_t(i) Y_t(i) - (1-\tau)\frac{W_t}{Z_t} Y_t(i) - \frac{\gamma}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t Y_t \right]$$

where $Q_{t,t+j} = \beta^j \prod_{s=1}^{j}(1 + r^n_{t+s-1}) \frac{C_{t+j}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+j}}$ is used to evaluate future nominal cash flows.

Substituting the demand function for output and taking first order conditions yields the optimal reset price,

$$\begin{aligned} 0 = &\left[(1-\sigma)\left(\frac{P_t(i)}{P_t}\right)^{-\sigma} + \sigma(1-\tau)\frac{W_t}{P_t}\frac{1}{Z_t}\left(\frac{P_t(i)}{P_t}\right)^{-\sigma-1} - \gamma\frac{P_t}{P_{t-1}(i)}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)\right] Y_t \\ &+ \mathbb{E}_t Q_{t,t+1}\left[\gamma \frac{P_{t+1}P_{t+1}(i)}{(P_t(i))^2}\left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right)\right] Y_{t+1} \end{aligned}$$

Since this problem is identical for each firms, they all charge the same price $P_t(i) = P_t$. Defining the gross inflation rate $\Pi_t = P_t/P_{t-1}$, the expression simplifies,

$$0 = \left[(1-\sigma) + (\sigma - 1)\frac{W_t}{P_t}\frac{1}{Z_t} - \gamma \Pi_t (\Pi_t - 1)\right] Y_t + \beta(1 + r^n_t)\mathbb{E}_t \frac{C_{t+1}^{-1}}{C_t^{-1}}\left[\gamma \Pi_{t+1}(\Pi_{t+1} - 1)\right] Y_{t+1}$$

$$\Pi_t(\Pi_t - 1) = \frac{\sigma - 1}{\gamma}\left[\frac{W_t}{P_t}\frac{1}{Z_t} - 1\right] + \beta(1 + r^n_t)\mathbb{E}_t\left[\frac{C_t}{C_{t+1}}\frac{Y_{t+1}}{Y_t}(\Pi_{t+1} - 1)\Pi_{t+1}\right].$$

Define $\kappa^* \equiv \frac{\sigma - 1}{\gamma}$ to get the equation in the text.

A.3 Government The central bank sets the nominal interest rate according to an interest rate rule subject to zero lower bound constraint,

$$1 + i_t = \max\{\beta^{-1} + \phi(\Pi_t - 1), 1\}$$

Any subsidies to firms and any government spending is financed by lump-sum taxes within the period,

$$T_t = \tau \frac{W_t}{P_t} N_t + G_t.$$

Thus, the government runs a balanced budget each period.

Steady-state government spending is $\bar{G} = s_g \bar{Y}$.

A.4 Market clearing All markets clear if and only if

$$
\begin{aligned}
L_t &= N_t, \\
C_t + G_t + \frac{\gamma}{2} (\Pi_t - 1)^2 Y_t &= Y_t, \\
B_t &= 0.
\end{aligned}
$$

A.5 Steady-state We define the steady-state as the state of the economy without shocks and zero inflation:

$$
\begin{aligned}
\bar{L} &= \bar{N}, \\
\bar{G}(i) + \bar{C}(i) &= \bar{Y}(i), \\
\bar{C} + \bar{C} &= \bar{Y}, \\
\bar{B} &= 0, \\
\bar{i} &= \beta^{-1} - 1, \\
\bar{\pi} &= 0, \\
\bar{MC} &= 1 \\
\bar{Z} &= 1, \\
\frac{\bar{W}}{P} &= 1, \\
\bar{Y} &= \left(\frac{1}{\chi(1 - s_g)} \right)^{\frac{1}{1+\psi}}, \\
\bar{L} &= \bar{Y}, \\
\bar{T} &= \frac{1}{\sigma} \bar{Y} + s_g \bar{Y} \\
\bar{\xi} &= 1
\end{aligned}
$$